

Multiresolution Analysis

Basis of translates

$$V_0 = \text{span}\{\varphi(x - k)\}_{k \in \mathbb{Z}} \quad W_0 = \text{span}\{\psi(x - k)\}_{k \in \mathbb{Z}}$$

Translation and Dilation

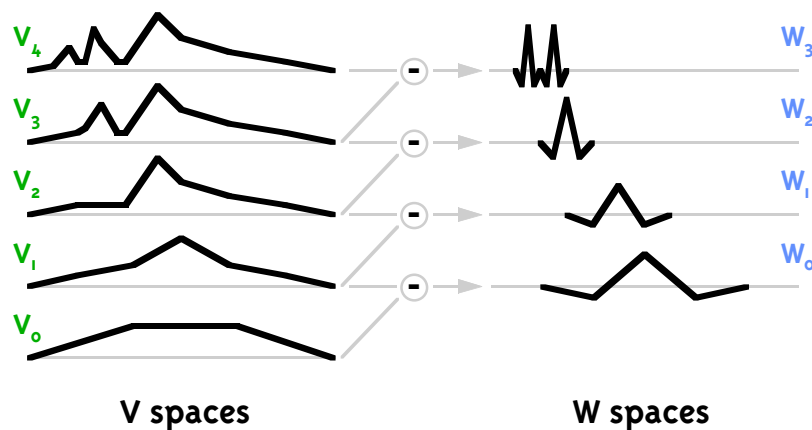
$$f(x) \in V_j \Rightarrow f(x+1) \in V_j \quad f(x) \in V_j \Rightarrow f(2x) \in V_{j+1}$$

Ladder of spaces

$$V_j \subset V_{j+1} \quad \bigoplus_{j \in \mathbb{Z}} W_j = L_2(\mathbb{R})$$

$$V_{j+1} = V_j \oplus W_j$$

Multiresolution Analysis



Other Classic Constructions

Daubechies

- orthogonal wavelets of arbitrary smoothness

Tons of others...

Lifting

- any finite filter



Other Properties

Orthogonality

- preserves energy
- bi-orthogonality more flexible

Smoothness

- localization towards high frequencies

Vanishing moments

- localization towards low frequencies

Stability

Web Resources

Wavelet digest

- <http://www.math.sc.edu/~wavelet>
 - newsletter, search engine, bibs, software

Amara's wavelet page

- <http://www.amara.com/current/wavelet.html>
 - short intro to wavelets
 - overview of many software packages
-

5

Web Resources

Mathsoft wavelet resources

- <http://www.mathsoft.com/wavelets.html>
 - links to many online papers

Course page

- <http://www.cs.caltech.edu/~ps/waveletcourse/>
 - online course materials and these slides
 - liftpack
-

6

Books on Wavelets

Wavelets and Subband Coding

- Vetterli/Kovacevic
- signal processing point of view
 - <http://cm.bell-labs.com/math/people/jelena/book.html>

Wavelets for Computer Graphics: Theory and Applications

- Stollnitz/DeRose/Salesin
 - <http://www.amath.washington.edu/~stoll/pub.html#WaveletBook>
-

7

Books on Wavelets

Wavelets and Filter Banks

- Strang/Nguyen
 - combines math and signal processing: very accessible with many exercises and Matlab toolbox
 - <http://saigon.ece.wisc.edu/~waveweb/Tutorials/book.html>
-

8