CS/CNS 257c: Simulation
Due Monday, May 10, 11:59pm, *electronic submission*

Homework 4

1. **FE approximation for the convection diffusion equation.**
   Implement the elemental stiffness matrix and load vector for a convection diffusion element. As discussed in class, for stability reasons you need to use the Petrov-Galerkin method for the discretization. Use your existing finite element code for the assembly of the global stiffness matrix and load vector.

   Verify your implementation with the following test problem:

   $$50\ \theta_x - \theta_{xx} = 0$$

   with Dirichlet boundary conditions:

   $$\theta(0) = 0 \quad \text{and} \quad \theta(10) = 1$$

   Use ten linear elements for the discretization. Visualize the temperature with the following stability parameters: $\alpha = 0$, $\alpha = \alpha_{opt}$, and $\alpha = 1$.

2. **Non-stationary FE approximation for the convection diffusion equation.**
   Use your finite element program from part 1 to solve the nonstationary convection diffusion problem in one dimension. The time integration of the semidiscrete equations can be done with the implicit Euler scheme. As initial conditions choose the following temperature distribution:

   $$\theta(x,0) = \begin{cases} sin(\pi x) & 0 < x < 1 \\ 0 & 1 < x < 10 \end{cases}$$

   The temperature at the left boundary is given by $\theta(0,t) = 0$. The heat flux at the right boundary is given by $\theta_x(10,t) = 0$. The convective velocity is 50. Due to the concentrated initial conditions, use a fine mesh with 200 elements for your computations.

3. **Extra credit**
   Add mesh refinement and unrefinement to your program. The nonstationary case can be handled much better if the mesh is adaptively refined dependent on the position of the wave front. Apply your adaptive finite element program to the nonstationary convection diffusion equation in the second example.