CS/CNS 257c: Simulation
Due Monday, May 10, 11:59pm *with homework 3, or seperate*

Homework 3 - Update

1. **Implement the PBCG algorithm.**
   There was an error in the algorithm presented for positive definite matrices. To implement PBCG correctly, make the following corrections:

$$z_{k+1} = Br_{k+1}$$
$$\overline{z}_{k+1} = B^T\overline{r}_{k+1}$$

   For this problem, you were expected to implement BCG that solves $Ax = b$ when A is one of the following: symmetric, positive-definite, or symmetric-positive-definite.

   If the previous typo prevented you from finishing the assignment on time, please contact one of the TA's immediately. Otherwise, we will make the above modification to your code and grade that version.

2. **Time integration.**
   The intent of this problem was not clear as originally stated; below is the revised version. You have the option of turning in a revised solution based on the revised problem. We will grade based on the revised problem. (This is a short problem and should not take very long.)

   In this problem we will investigate the **time stability** of the backward, forward and trapezoidal integration methods. These methods all deal with the general differential equation: $y'(t) = f(t)$.

$$\begin{array}{ll} \text{(forward Euler)} & y(t+\Delta t) = y(t) + \Delta t f(t) \\ \text{(backward Euler)} & y(t+\Delta t) = y(t) + \Delta t f(t+\Delta t) \\ \text{(trapazoidal rule)} & y(t+\Delta t) = y(t) + \Delta t \frac{f(t)+f(t+\Delta t)}{2} \end{array}$$

   Let's specialize to the case $y'(t) - a\ y(t) = 0,\ y(0) = 1$. Here a is complex and $Re(a) \leq 0$. Notice that the analytic solutions are bounded as $t \to \infty$.

   For what (complex) values $z = a\Delta t$ will the discretized solution $y^n := y(n\Delta t)$ stay bounded as $n \to \infty$? Your answer should include drawings of three regions in the complex plane, one region per discretization method.

   Comment a little on why these regions are important, how the different methods compare to one another, and the meaning of the above analysis when $Re(a) > 0$.